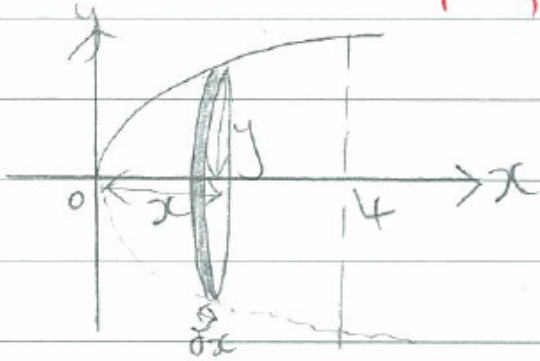


M3 - June 2006

1-



$$\pi \int_a^b y^2 x dx = \pi \int_0^4 y^2 dx \times \bar{d}$$

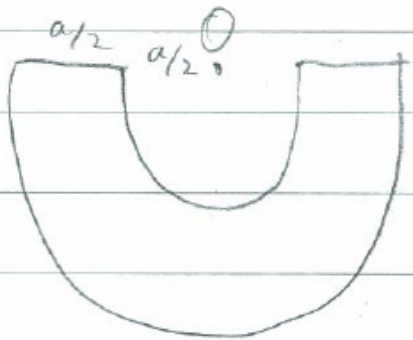
$$\int_0^4 x^2 dx = \int_0^4 x dx \times \bar{d}$$

$$\frac{1}{3} [x^3]_0^4 = \frac{1}{2} [x^2]_0^4 \times \bar{d}$$

$$\frac{64}{3} = 8\bar{d}$$

$$\bar{d} = \frac{8}{3}$$

2-



$$a) \frac{3}{8} a \cdot \frac{2}{3} a^2 - \frac{3}{8} \cdot \frac{a}{2} \cdot \frac{2}{3} a^2 = \left(\frac{2}{3} a^3 - \frac{2}{3} a^3 \right)$$

$$\frac{1}{4} a - \frac{1}{64} a = \frac{7}{12} a$$

$$\bar{d} = \frac{45}{112} a$$

$$b) \frac{45}{112} m + \frac{3}{8} \cdot \frac{1}{2} \cdot km = (m+km) \cdot \frac{17}{48}$$

$$\frac{45}{112} m + \frac{3}{16} km = \frac{17}{48} m + \frac{17}{48} km$$

$$\frac{1}{21} m - \frac{1}{6} km = 0$$

$$m \left(\frac{1}{21} - \frac{1}{6} k \right) = 0$$

$$k = \frac{6}{21} = \frac{2}{7}$$

$$3- a) f = 5, r = 0.1, \omega = 2\pi f = 10\pi$$

$$a_{\max} = \omega^2 r = (10\pi)^2 \times 0.1 = 10\pi^2$$

$$[F = ma]$$

$$F_{\max} = 0.2 \times 10\pi^2 = 19.7 \text{ N (3sf)}$$

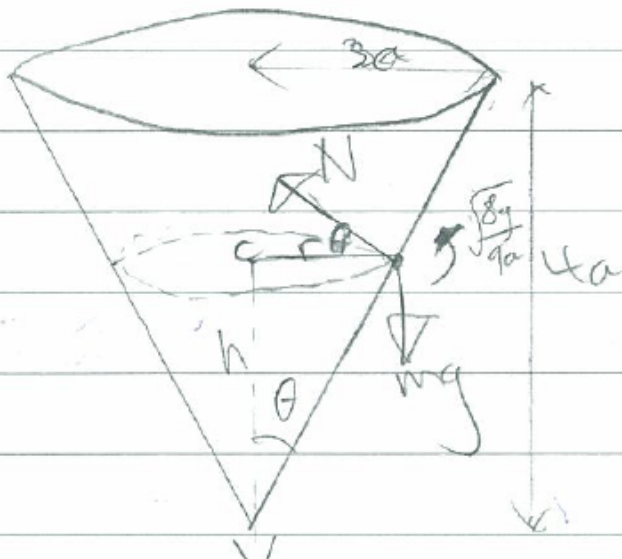
$$b) f = 5, r = 0.2 \quad w = 2\pi f = 10\pi$$

~~$$v^2 = w^2(a^2 - x^2) = 10^2 \pi^2 (0.2^2 - 0.1^2)$$~~

$$v^2 = w^2(a^2 - x^2) = 10^2 \pi^2 (0.2^2 - 0.1^2)$$

$$v = 5.44 \text{ ms}^{-1} \text{ (3sf)}$$

4.



$$[F = ma]$$

$$N \cos \theta = m w^2 r$$

$$N \sin \theta = \frac{m g \cdot r}{9a}$$

$$N \cdot \frac{4}{5} = \frac{m g \cdot r}{9a}$$

$$r = \frac{9a N}{10mg} \quad \text{①}$$

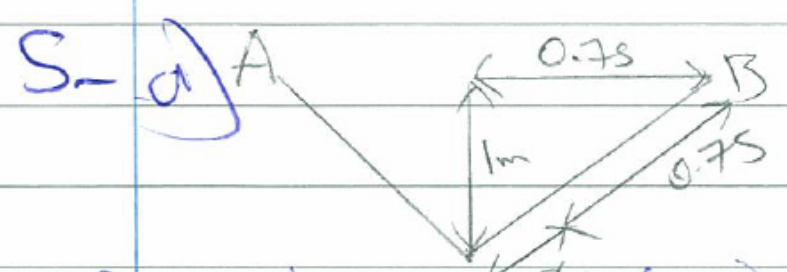
$$\uparrow mg = N \sin \theta = N \cdot \frac{3}{5}$$

$$N = \frac{5mg}{3} \quad \text{②} \quad \text{③} \quad \text{④} \quad r = \frac{9a \cdot \frac{5mg}{3}}{3 \times 10mg} = \frac{3a}{2}$$

$$\tan \theta = \frac{r}{h} = \frac{3}{4}$$

$$\frac{3a}{2h} = \frac{3}{4}$$

$h = 2a$ \therefore Height of C above V is $2a$

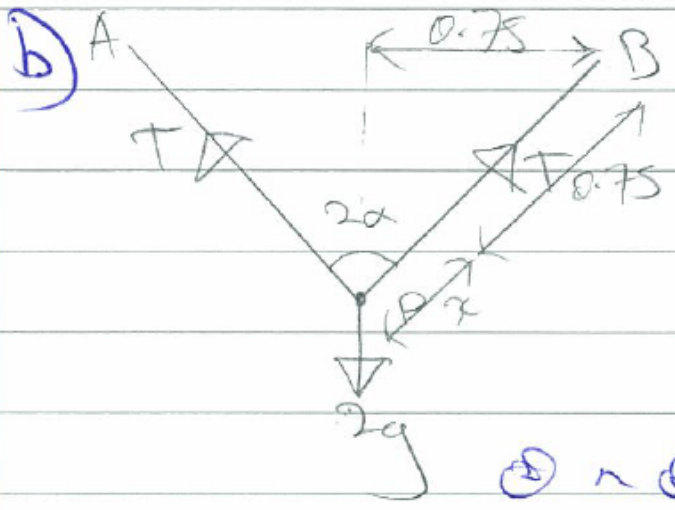


$$0.75 + x = \sqrt{1^2 + 0.75^2} = 1.25$$

$$x = 0.5$$

~~$$y \times 2 = \frac{1}{2} \times 2 \times v^2 + \frac{49 \times 0.5^2}{2 \times 0.75} \times 2$$~~

$$v^2 = 2g - 16\frac{1}{3} \quad v = 1.81 \text{ ms}^{-1} \text{ (3sf)}$$



$$\uparrow 2T \cos \alpha = 2g$$

$$T \cos \alpha = g \quad (1)$$

$$T = \frac{4g}{a} = \frac{4g \alpha}{0.75}$$

$$\textcircled{2} \text{ \& } \textcircled{1} \quad 196x \cos x = 3g \quad x = \frac{3}{20 \cos x} \quad (2)$$

$$20x \cos x = 3$$

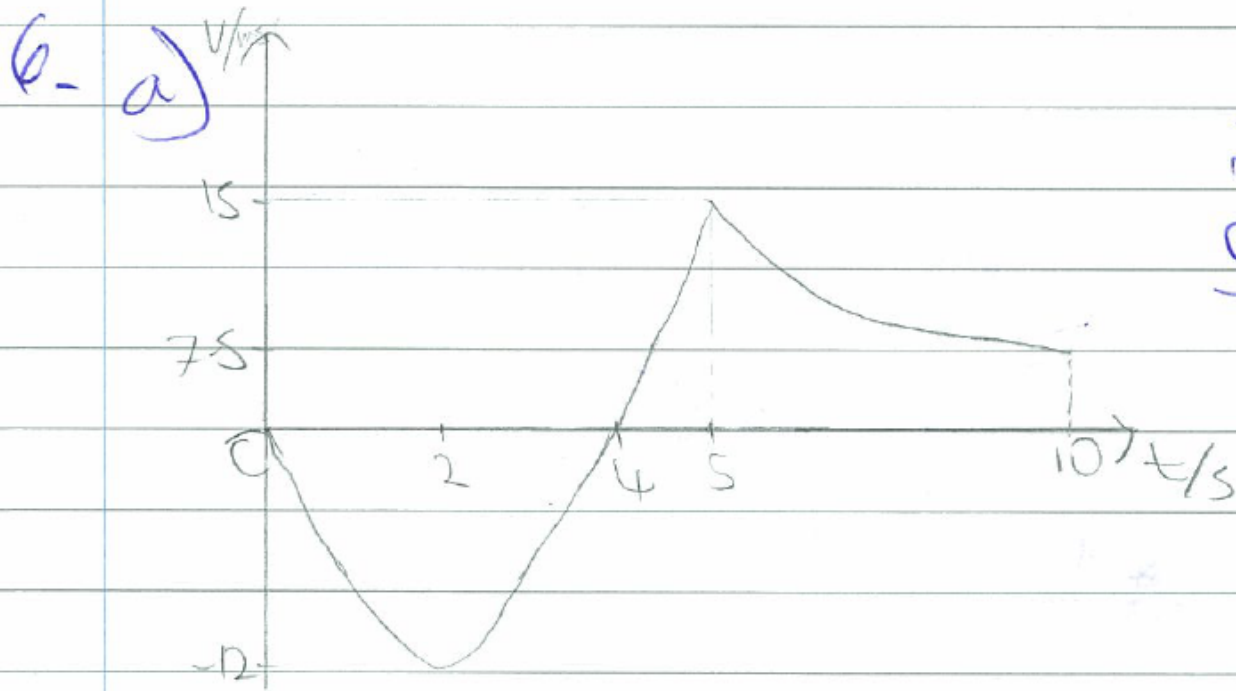
$$\sin \alpha = \frac{0.75}{0.75 + x} \quad (4)$$

$$\textcircled{2} \text{ \& } \textcircled{3} \quad \sin \alpha = \frac{0.75}{0.75 + \frac{3}{20 \cos \alpha}}$$

$$\frac{3}{4} \sin \alpha + \frac{3 \sin \alpha}{20 \cos \alpha} = \frac{3}{4}$$

~~$5 \sin \alpha + 3 \tan \alpha = 5$~~

$$5 \sin \alpha + 3 \tan \alpha = 5$$



b) $2 < t < 5$

c) Distance = $\int_0^4 3t^2 - 2t \, dt + \int_4^5 3e^{-2t}$
 $= -9 \left[\frac{1}{3}t^3 - 2t^2 \right]_0^4 + 3 \left[\frac{1}{3}t^3 - 2t^2 \right]_4^5$
 $= -9 \left(\frac{64}{3} - 32 \right) + 3 \left(\frac{125}{3} - 150 - \frac{64}{3} + 32 \right)$
 $= \cancel{32} + 7 = 39$

i) Distance in 4 seconds = 32 m

$32 - 7 = 25 \text{ m}$

$25 = \int_0^t 75t^{-1} \, dt$

$\ln t = \frac{1}{3} + \ln 5$

$\frac{1}{3} = (\ln t)_5$

$t = 6.985 \text{ (3sf)}$

$\frac{1}{3} = \ln t - \ln 5$

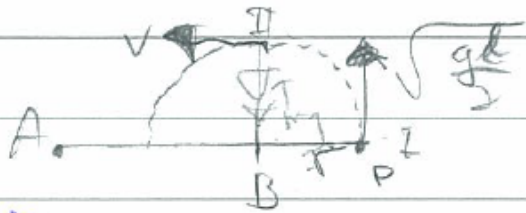
$$7. a) \frac{1}{2} \cdot m \cdot \frac{5gl}{2} = m \cdot g \cdot l + \frac{1}{2} m v^2 \cdot 2$$

$$5gl = 4gl + 2v^2$$

$$v^2 = \frac{gl}{2}$$

$$v = \sqrt{\frac{gl}{2}}$$

b)



$$ME_{\#} = ME_I$$

$$\frac{1}{2} \cdot m \cdot \frac{gl}{2} = mgr + \frac{1}{2} m v^2$$

$$\frac{gl}{4} = gr + \frac{v^2}{2}$$

$$v^2 = \frac{gl}{2} - 2gr$$

$$[F = ma]$$

$$T + mg = \frac{m}{r} \left(\frac{gl}{2} - 2gr \right)$$

$$T + mg = \frac{mgl}{2r} - 2mg$$

$$T = \frac{mgL}{2r} - 3mg$$

For complete semicircle, $T \geq 0$

$$\frac{mgL}{2r} - 3mg \geq 0$$

$$\frac{L}{2r} \geq 3$$

$$L \geq 6r \quad \text{①}$$

$$AB = L - r$$

~~AB = L - r~~

$$r = L - AB \quad \text{②}$$

~~AB = L - r~~



$$\begin{aligned} L &\geq 6r - BAB \\ BAB &\geq \frac{L}{3} \\ AB &\geq \frac{L}{6} \end{aligned}$$